NOTATION

ρ_i, u_i, v_i, E_i, density, velocity components along the z and r axes, and internal energy of i-th phase, respectively; p, gas pressure; c_p, specific heat of gas at constant pressure; R, particle radius; θ, angle of rotation of particle separatrix; c_D, drag coefficient of spherical particle; ρ_0, density of particle material; r_0, nozzle radius, m; γ, adiabatic modulus; n, underexpansion of nozzle; ρ^x, u^x, v^x, p^x, gas parameters at side wall of cell; β_p, angle of rotation of side wall of cell with respect to axis; M, mach number of jet or cotraveling flow.

LITERATURE CITED

1. V. I. Blagosklonov and A. L. Stasenko, Uch. Zap. TsAGI, 8, No. 1, 32-42 (1977).
2. V. I. Blagosklonov, M. M. Glinskii, and A. L. Stasenko, in: Jet and Breakaway Flows [in Russian], Moscow (1979), pp. 95-105.
3. G. A. Saltanov, Nonequilibrium and Nonsteady Processes in Gas Dynamics [in Russian], Moscow (1970).
4. V. I. Kopchenov and A. N. Kraiko, Tr. Inst. Mekh. Mosk. Gos. Univ., No. 32, 96-108 (1974).
5. S. K. Godunov (ed.), Numerical Solution of Multidimensional Gas-Dyanmic Problems [in Russian], Moscow (1976).
6. D. Karlson and R. Khoglund, Raket. Tekh. Kosmonavt., 2, No. 11, 104-109 (1964).
7. L. D. Landau and E. M. Lifshits, Mechanics of Continuous Media [in Russian], Moscow (1954).
8. R. Courant, K. O. Friedrichs, and H. Lewy, Math. Ann., 100, 32-74 (1928).
9. G. Mairels and Dzh. Mullen, Raket. Tekh. Kosmonavt., 1, No. 3, 65-72 (1963).
10. K. P. Stanyukovich, Nonsteady Motion of a Continuous Medium [in Russian], Moscow (1971).

REGULAR AND STOCHASTIC DYNAMICS OF PARTICLES DURING VORTEX GENERATION IN A ROTATING STREAM WITH SHEAR

E. V. Guslyakova and A. A. Solov'ev

UDC 532.517.4

The results of an experimental study of the initial stage of development of a concentrated vortex are given.

Many practical problems of thermophysics and hydrodynamics require knowledge of the laws of organization of motions in vortex formations [1, 2]. There are a number of ways of exciting vortices [3], but it still remains unclear what physical processes occur in the initial stages of their formation. The mechanism of formation of concentrated vorticity in the presence of a trigger disturbance is well known [3]. During the further evolution of the initial disturbance and its conversion into a vortex, the character of the particle motion remains regular. A generation mechanism of an entirely different type, when a concentration of vorticity arises from random motions of particles, is possible in principle [4-7]. This regime of excitation of vortex formations has hardly been studied.

An installation was built to obtain and investigate vortices: a rotating cylindrical chamber, the flat bottom of which consists of a disk and two concentric rings [8]. The diameter of the installation is 0.46 m. The disk has a radius of 0.1 m, while the middle and outer rings have widths of 0.1 and 0.03 m, respectively. Water, rotating together with the vessel, fills it to a depth of 0.025 m. The liquid is subjected to the action of two opposite flows. They are created by clockwise rotation of the disk and counterclockwise rotation of both rings. The rings rotate at the same velocity, different from the velocity of the disk. The vessel as a whole rotates at the same velocity and in the same direction as the disk. The sign of rotation of the rings is arbitrarily taken as negative. The motions at the surface of the liquid were made visible by light scattering from microparticles moving along with the

State Oceanographic Institute, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 1, pp. 37-42, July, 1987. Original article submitted April 1, 1986.

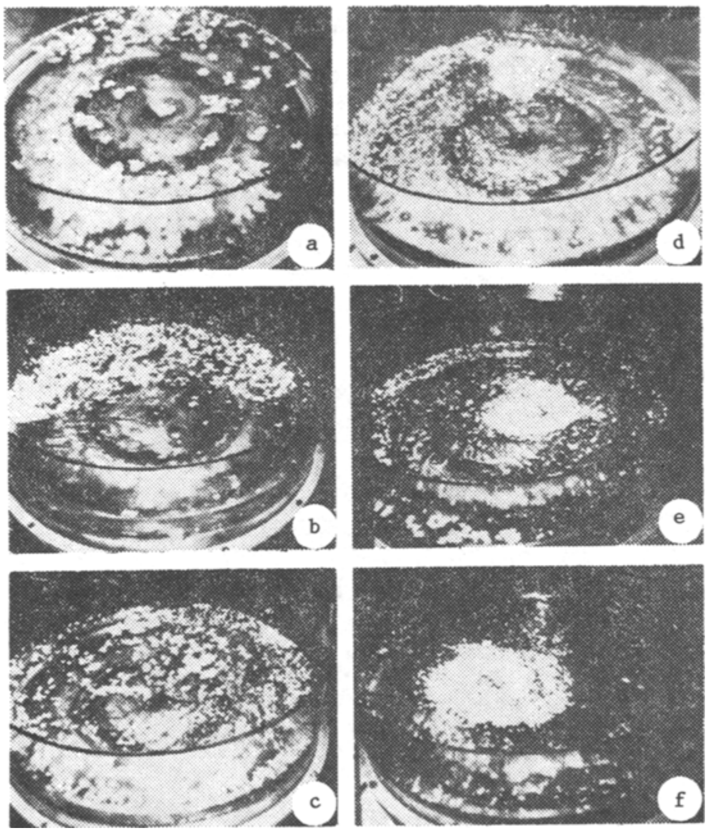

Fig. 1. Three-dimensional structures excited in a rotating liquid at certain values of the Rossby (Ro) and Taylor numbers (Ta, 10^7) and of the parameter of the transition from state to state (ξ_{ij}, 10^7): a) Ro = 1.0, Ta = 1.33; b) Ro = 0.83, Ta = 3.08, ξ_{ab} = 0.17; c) Ro = 0.69, Ta = 6.9, ξ_{bc} = 6.0; d) Ro = 0.71, Ta = 27.8, ξ_{bd} = 0.2; e) Ro = 0.62, Ta = 35.7, ξ_{ed} = 0.4; f) Ro = 0.64, Ta = 50, ξ_{re} = 2.0.

stream. Particles with a size of 10-20 μm were used for velocity measurements from trajectories of their motion recorded on photographic film. Qualitative observations of the organization of motions during the excitation of vortex formations were made with particles of polyurethane foam with a size of 200 μm. The methods of observation and analysis of the patterns formed at the liquid surface are similar to those used in a number of papers [9-11]. The experiments were made with different ratios between the angular velocities of rotation of the disk and rings.

The initial steady three-dimensional structure which is excited in the rotating liquid consists of two elliptical cells. They develop upon a relatively small difference, fixed in time, in the rotation velocities of the disk and rings (Fig. 1a). The motion of particles in these cells has a closed character. With an increase in the difference between the rotation velocities of the disk and rings, the two cells start to be transformed into one. It has the form of a steady in time, spatially curved, broad, light band. A further increase in the difference between the angular velocities of the disk and rings leads to the final disappearance of the two cells (Fig. 1b).

The character of the evolution of a single cell depends essentially on the amounts by which the velocity shear and the overall rotation increase in the given stage of development of the motion in comparison with the preceding stage.

If one establishes a disk-ring shift in angular rotation considerably different from the value at which a single cell develops, while the overall rotation is increased very little, then the following pattern is observed. The clearly defined edges of the cell are blurred and the circulations are destroyed. The particle motions ultimately acquire an entirely random character (Fig. 1c).

In the case when successive values of the velocity shear differ relatively little from the preceding ones while the overall rotation changes considerably, other transitions occur with a cell. It is not destroyed, and only the character of the motions in its different parts changes. A surge (source) of liquid is formed at one end of the cell while a drop (sink) is formed at the other end. In this case the mass of particles making the structure visible acquires the shape of a comma (Fig. 1d). The focal point of convergence of the strip covered by particles initially has an insignificant size.

With an increase in the radial gradient of vorticity of the working liquid, the following occurs. The size of the region to which particles are drawn at an angle to the curved strip becomes larger. The "density" of the strip, or the number of particles in it, decreases. The sharpness of the boundary of the strip is more clearly displayed, while the angle at which the strip approaches the central cluster of particles increases. The rotation of the cluster of particles has a positive sign. The circulation of particles in the strip is in the opposite direction.

For quantitative estimates of the discrete changes in the shear velocity and the velocity of overall rotation at which the modification of structures occurs, we used the Rossby and Taylor numbers. The transition between structures was characterized by the functional dependence of the Rossby number on the inverse Taylor number: $Ro = f(Ta^{-1})$. A structure of the type of two cells is converted into one cell if the size $\Delta(Ro)$ of the change in the Rossby number between these states, divided by the change $\Delta(Ta^{-1})$ in the inverse Taylor number, has a value $\xi_{ab} = 0.17 \cdot 10^{7}$.

As a result of numerous experiments, it was established that random motions of particles are replaced by organized ones for $\xi_{ce} = 0.40 \cdot 10^{7}$ with $0.64 < Ro < 0.67$ and $35 \cdot 10^{-10} < Ta^{-1} < 110 \cdot 10^{-10}$ with a good degree of duplication. A regular structure is formed from irregular motions with a decrease in the Rossby number and an increase in the Taylor number in this range. It has the form of a round cluster of particles with irregular edges. Within the cluster there is rotation differing in velocity from the motion of the surrounding liquid. A clearly expressed region free of particles in the center of the cluster is subsequently formed. It corresponds to the core or so-called "eye" of a vortex formation (Fig. 1e).

From Fig. 2 it follows that, by assigning values of Ro and Ta^{-1} in accordance with the equation $Ro = 3.7 \cdot 10^{13}\ Ta^{-1} + 0.5$, one can obtain a vortex from irregular particle motion. We call this regime of generation stochastic, having in mind that in this case the dissipative system differs from total chaos by the capacity for incipient stochasticity. Another regime of vortex generation with regular particle motion is also determined by a linear dependence of Ro and Ta^{-1} in the form $Ro = 6.6 \cdot 10^{13}\ Ta^{-1} + 0.5$. By comparison with the preceding case, this regime of vortex generation is characterized by a larger value of the angular coefficient of the linear dependence $Ro = f(Ta^{-1})$.

For values of the parameter $\xi_{df} = 2.0 \cdot 10^{7}$ in the range of $0.66 < Ro < 0.68$ and $22 \cdot 10^{-10} < Ta^{-1} < 31 \cdot 10^{-10}$, the development of vorticity with an "eye" at the center of the installation occurs as a result of the concentration of rotation in a spiral cell. A region of trapped particles rotating about the focal point of convergence of a spiral strip appears initially. Then particles from the spiral strip flow over entirely into the round cluster. A vortex that was formed through the regular motion of particles differs little in external features, in the final phase of its development, from one formed from random motions (cf. Fig. 1e and f).

In both cases, the radius of the vortex core is less than the Rossby–Obukhov radius, while the characteristic external size is greater than this size. It must be mentioned that before a vortex moves to the center of the vessel, it drifts in the direction of overall rotation of the system, lying on a line of velocity shear. In both generation regimes its drift velocity is close to the characteristic Rossby velocity.

Now let us dwell on a comparison of the vortex formations that are generated from regular and random motions of particles. To estimate the distribution of vorticity in the vortex formations obtained, we use a method of analysis of the profiles of the funnels formed below a vortex. The observed profile of the free surface of the liquid was corrected with allowance for surface tension [12]. As it turned out, the velocity distribution can be approximated with a sufficient degree of accuracy by the law

$$V = (\omega + \Omega),\ 0 \leqslant r \leqslant r_0;$$

$$V = \frac{A}{r^n} + \Omega r,\ r \geqslant r_0.$$

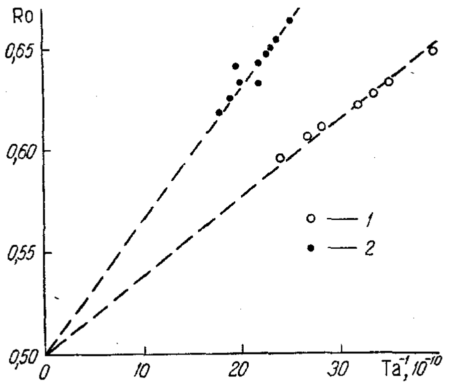

Fig. 2

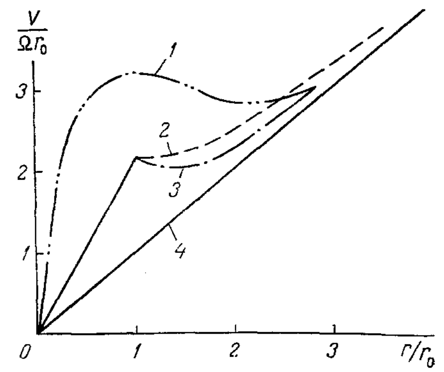

Fig. 3

Fig. 2. Dependences Ro = $f(Ta^{-1})$ corresponding to regimes of generation from random (1) and regular motions (2).

Fig. 3. Velocity distribution in the vortex relative to the average rotation of liquid in the vessel: 1) profile obtained for the vortex by Hopfinger and Browand [12]; 2, 3) profiles for regimes of vortex generation from regular and random motions, respectively; 4) velocity distribution of the average rotation of the entire mass of liquid in the vessel.

TABLE 1. Radial Distributions of the Coefficient of Turbulent Viscosity ν', 10^{-4} m^2/sec, in Vortex Formations Arising from Regular (I) and Random Particle Motions (II)

r/r_0	I	II
1,0	10,4	14,1
1,2	9,0	12,5
1,4	7,0	11,4
1,6	5,9	10,5
1,8	5,2	9,7
2,0	4,5	9,1
2,5	3,5	7,9
3,0	2,8	7,0

The exponent of the power law of radial velocity variation was determined from the ratio of the finite-difference radial velocity gradient to the angular velocity of rotation of the vortex:

$$n = \frac{\Delta V}{\Delta r} \Big/ \frac{V_0}{r_0}.$$

If we assume that $\Delta V \approx V_0$ while $\Delta r = R - r_0$, then $n = x/(1 - x)$, where $x = r_0/R$. From Fig. 3 it is seen that a concentration of vorticity, exceeding the initial value set by the overall rotation of the system, is noted in the laboratory experiments and in the tests of [12]. Here the intensities of the vortex formations generated in the regular and the stochastic regimes can be considered as practically the same. There is a difference between them, as can be seen from Fig. 3, only in the degree of the gradient nature of the radial profiles of tangential velocity. The latter is a reflection of the different action of turbulence on the initial vorticity. Let us calculate the radial distribution of the coefficient of turbulent viscosity for the two regimes of vortex generation. We use the Kármán hypothesis [13], extending it to the case of plane rotation of an incompressible liquid in the form

$$\nu' = l^2 \left(\frac{dV}{dr} - \frac{V}{r} \right),$$

where

$$l = \varkappa \left(\frac{dV}{dr} + \frac{V}{r} \right) \Big/ \frac{d}{dr} \left(\frac{dV}{dr} + \frac{V}{r} \right)$$

For $V = A/r^n$ we have

$$\nu' = \frac{\varkappa^2 A}{(n+1) r^{n-1}} .$$

In Table 1 we give the data of calculations of ν' with $\varkappa = 0.35$ for the two regimes of vortex excitation. It is seen that in vortex formation during regular motion of particles the losses to turbulent friction prove to be smaller than in the case of the development of a vortex from chaos. One of the consequences emerging from this is the existence of a limit on the increase in average energy of the particles when their motion becomes stochastic. Vortex creation in a regime of regular particle motion is associated with relatively small losses to turbulent friction. This fact correlates well with observational data on the development of intense atmospheric vortices [14, 15].

NOTATION

ω_1, angular velocity of rotation of the disk ($\omega_1 > 0$); ω_2, angular velocity of rotation of the rings ($\omega_2 < 0$); $\Delta\omega = \omega_1 + \omega_2$, velocity chear; $\Omega = \omega_1 - \omega_2$, velocity of overall rotation of the liquid; $Ro = \Delta\omega/\Omega$, Rossby number; $Ta = \delta^4\Omega^2/\nu^2$, Taylor number; δ, filling depth of the rotating vessel with liquid; $\xi_{ij} = \Delta(Ro)/\Delta(Ta^{-1})$, parameter characterizing the transition from one structure to another; $r_R = \sqrt{g\delta}/\Omega$, Rossby–Obukhov radius; $V_R = \delta\Omega$, Rossby velocity; r_0, radius of the vortex "core"; R, external characteristic size of a vortex; n, exponent of the gradient variation of tangential velocity of a vortex; V_0, maximum velocity at the boundary of the vortex "core"; ω_0, angular velocity of rotation of the vortex "core"; ν', coefficient of turbulent viscosity; $\varkappa$, Kármán number; l, mixing length; A, circulation of a vortex.

LITERATURE CITED

1. A. V. Lykov, in: Problems of Heat and Mass Transfer [in Russian], Minsk (1976).
2. M. A. Gol'dshtik, Vortex Flows [in Russian], Novosibirsk (1981).
3. T. Maxworthy, in: Intense Atmospheric Vortices [Russian translation], Moscow (1985), pp. 260-284.
4. B. J. Cantwell, in: Vortices and Waves [Russian translation], Moscow (1984), pp. 9-79.
5. I. Prigozhin, From Existing to Originating: Time and Its Complexity in the Physical Sciences [in Russian], Moscow (1985).
6. O. G. Martynenko, A. A. Solov'ev, A. D. Solodukhin, et al., "Self-organization in turbulent vortex formations," Preprint No. 25, Inst. Teplo- Massoobmena, Akad. Nauk BSSR, Minsk (1984).
7. V. A. Bobr, L. Kh. Garmize, V. I. Kalilets, et al., in: Evolutionary Problems of Energy Transfer in Inhomogeneous Media [in Russian], Minsk (1982), pp. 3-19.
8. B. M. Lushin and S. S. Lappo, "A geohydraulic model," Inventor's Certificate No. 647572 USSR, Byull. Izobret., No. 6 (1979).
9. G. A. Antonova, B. P. Zhvaniya, D. G. Lominadze, et al., Pis'ma Zh. Eksp. Teor. Fiz., 37, No. 11, 545-548 (1983).
10. S. V. Antipov, M. V. Nezlin, and A. S. Trubnikov, Pis'ma Zh. Eksp. Teor. Fiz., 41, No. 1, 25-28 (1985).
11. E. V. Guslyakova, S. S. Lappo, and A. A. Solov'ev, in: Abstracts of Papers of the Second All-Union Symposium on Mechanisms of Generation of Small-Scale Turbulence in the Ocean [in Russian], Kaliningrad (1985), pp. 32-33.
12. E. J. Hopfinger and F. K. Browand, in: Intense Atmospheric Vortices [Russian translation], Moscow (1985), pp. 326-340.
13. T. Karman, in: Problems of Turbulence [Russian translation], Moscow–Leningrad (1936), pp. 271-286.
14. L. S. Minina, Practical Nephanalysis [in Russian], Leningrad (1970).
15. M. A. Herman, Space Methods of Research in Meteorology [Russian translation], Leningrad (1985).